Engineering Statics
in story and verse

by

Robin Ford

Statics made memorable

This review of engineering mechanics was inspired by the Rupert Bear Annuals of the 1950s.

For more about Rupert Bear see http://rupertbear.co.uk/index.html

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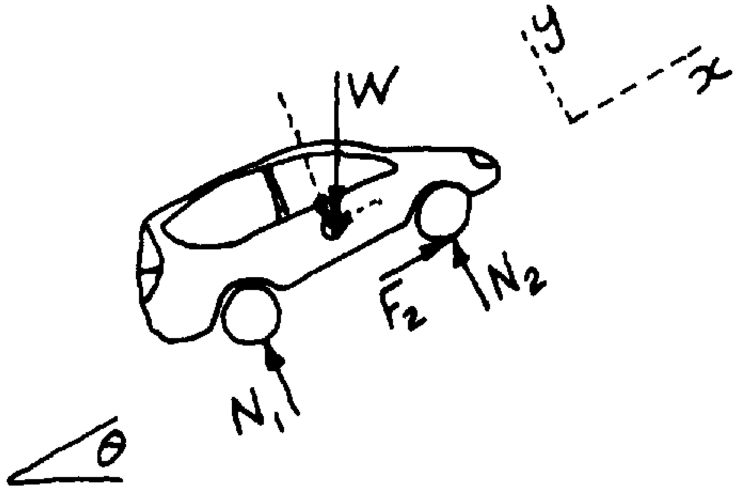
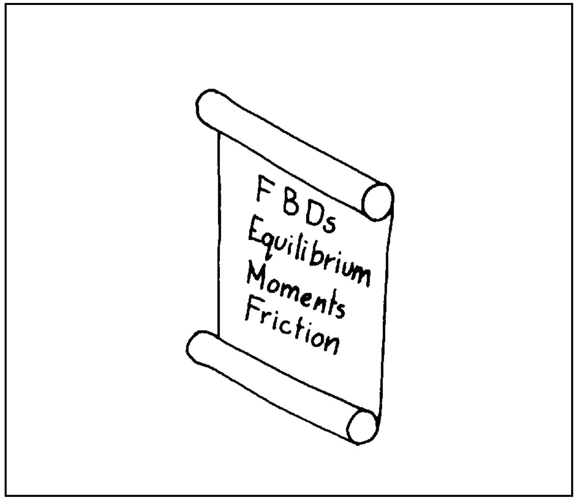
JAMIE AND THE CAR ON A HILL



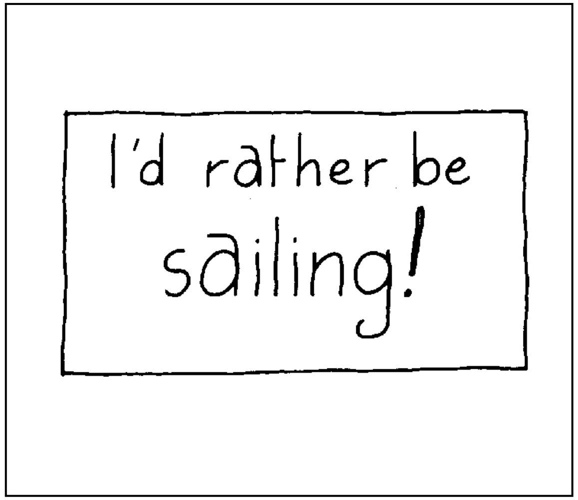
In which Jamie visits the Engineering Twins, learns some statics and is changed forever.

THE ENGINEERING TWINS PLAN TO TEACH JAMIE STATICS

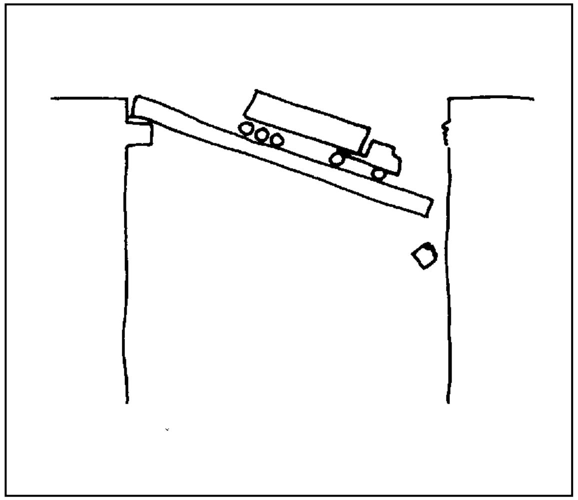
One sunny day, Jamie's friends the engineering twins are outside their shed working on their electric car project. As she bolts the new power controller into place, Lee says, "You know, Kim, Jamie could do with some engineering theory." "Definitely," replies Kim as he hands her the 12mm socket spanner, "Everyone could. Where would you start?" "I'd start with Engineering Statics," says Lee, and Kim agrees. "Free-body-diagrams, equilibrium, moments, friction – that sort of thing?" he suggests, thinking about how they had used statics for design calculations with the electric car. They'd found loadings that way, and used them in mechanics of solids analysis when working out how to make components strong enough. They'd used dynamics too (which builds on statics ideas) to understand vehicle performance and gears and mechanisms. They'd even used statics to understand the hydraulics for the brakes. "How can we educate Jamie without it seeming like a lecture," says Kim, "We can't just turn up and say, 'Hey, Jamie, you need to know about statics'." "We'll just have to wait our chance," says Lee.



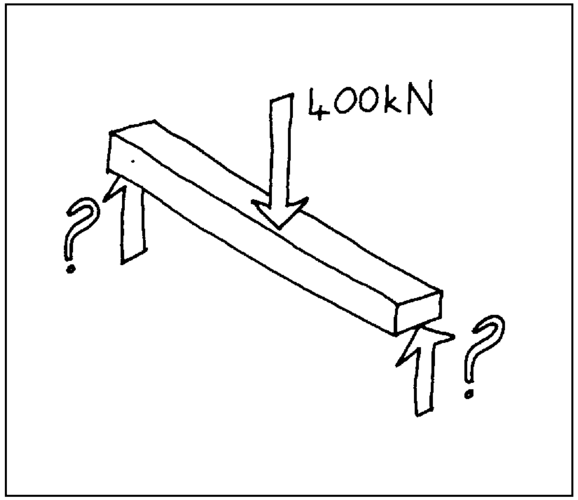
Twins Lee and Kim devise a plan:
"Teach Jamie Statics if we can."



The engineering twins next ask,
"What is the value in our task?"

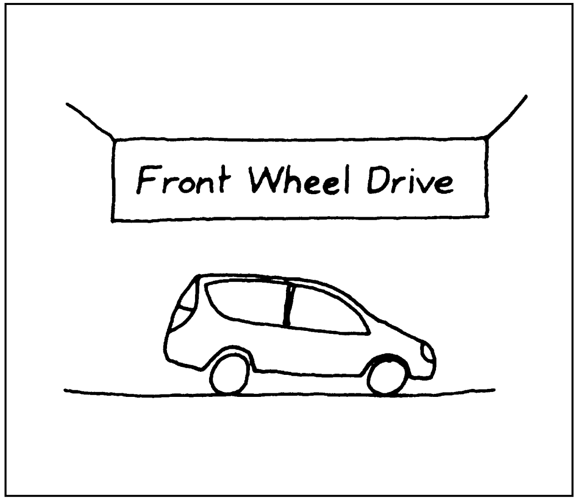


Their answer: "Statics helps you make
Quite sure that your designs won't break."

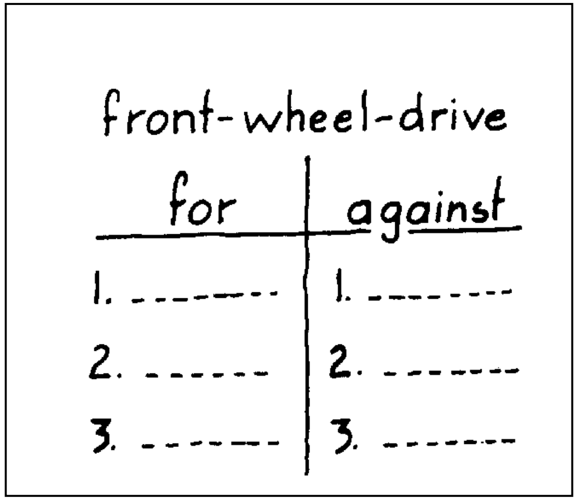


"It lets you know what loads apply,
How load is shared, and reasons why."

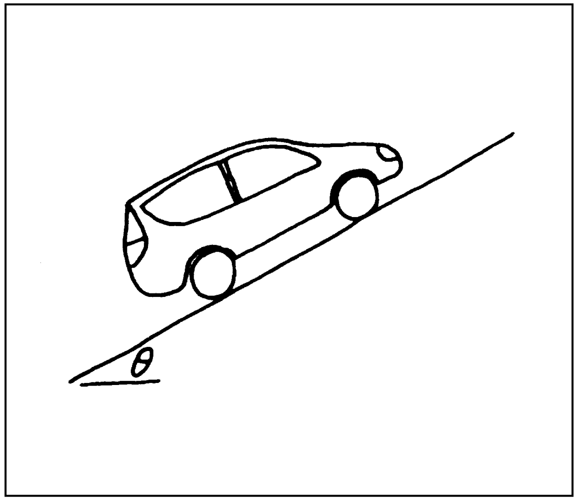
JAMIE WONDERS ABOUT A CAR'S HILL-CLIMBING ABILITY



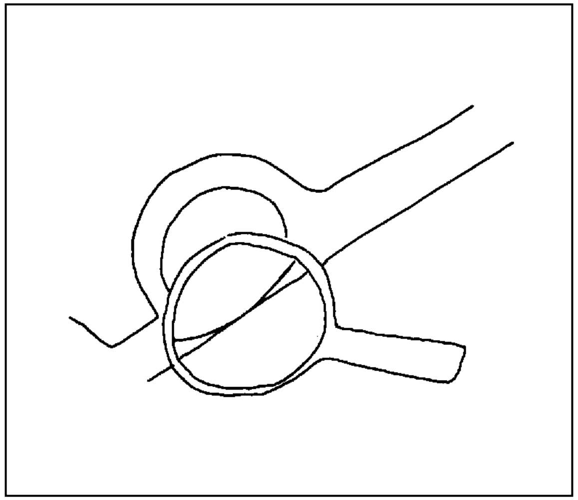
Their statics-teaching plan goes live
When Jamie buys a front wheel drive.



And Jamie asks them, "Do you know
If front wheel drive's the way to go?"



"Will rear weight-transfer mean less grip,
When climbing hills, and lead to slip?"

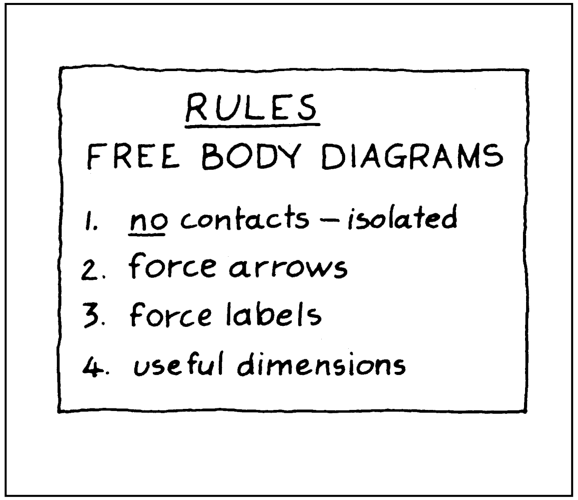


The twins smile, "Statics can reveal
The forces acting on each wheel."

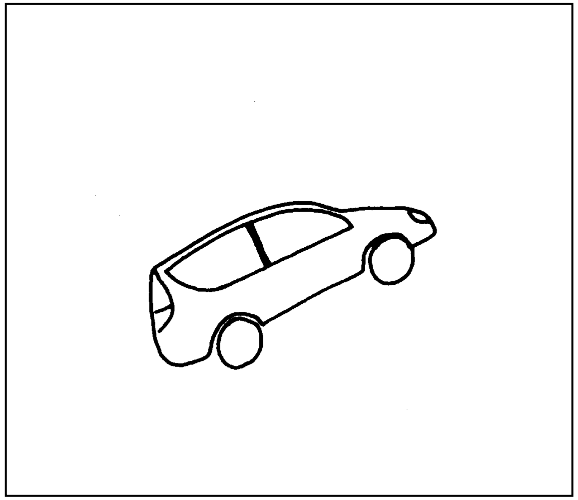
Jamie has a new car and is showing it to friends. They stand around and check out its features. "It's front-wheel-drive," says Jamie. After a moment's thought Pat says, "It might get wheelspin more easily than your old rear-wheel drive car when you drive up a slippery hill." Jamie asks why, and the friends explain, "It's weight transfer. On an incline weight is transferred from front to back so that the front driving wheels dig in less and the drive wheels have less grip." Jamie drives home wondering what this means for the gradients the car can handle on slippery gravel roads. "What's this weight transfer, anyway?" Jamie thinks, "And how can I work out the steepest hill my new car can climb? Of course! I'll stop by at the engineering twins, they'll know." Jamie finds them in their shed. "Hi, Jamie," says Lee "We were just talking about you." After they look at the new car, Jamie asks for their help. The twins smile at each other, get out a sketch-pad and begin to explain. "What you need to do," says Kim, "is work out the forces on each wheel as the car climbs the hill." "I can see that," says Jamie. "But how?"

JAMIE IS INTRODUCED TO FREE BODY DIAGRAMS

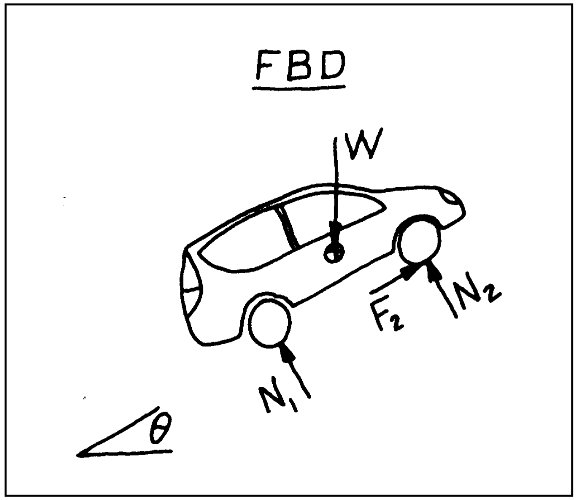
"First of all," says Kim, "you've got to know which forces you are interested in." "Fine. But what exactly is a force?" asks Jamie. "Well, it's a mystery," says Lee, "but forces describe how things interact – like tyres on a road. I think of them as big red arrows. Forces have a size, a direction, a line of action and a place where they apply." "Right," Kim continues, "So you decide which object you want to study and draw a picture that shows all the forces acting on it." "Do I just draw forces on a picture of the car on the hill?" Jamie asks. "No." says Kim with surprising force, "You've got to show it completely free of the ground. That's why it's called a *Free* Body Diagram, or FBD for short. Then the force arrows show how the ground interacts with the car." As Lee sketches the FBD Jamie asks, "What's that arrow pointing directly down near the middle of the car?" "It's the weight," says Kim. "You know, gravity pulling the car downwards." Jamie, uncertain, asks, "But how do you know where it acts?" Kim replies, with slight evasion, "It acts at the centre of gravity of course. Just assume that the car-maker can tell you where it is."



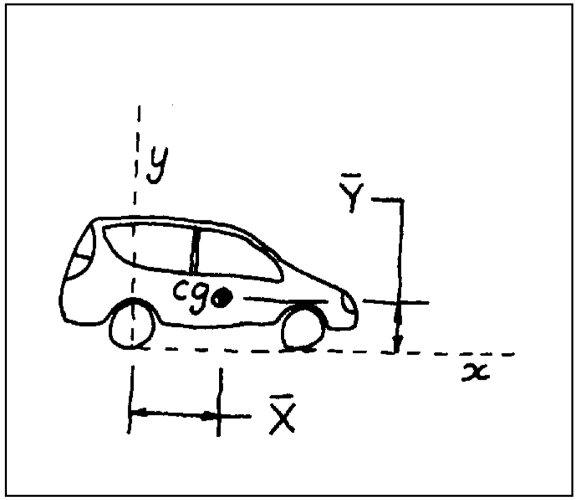
The twins start drawing carefully
A picture called an FBD.



They draw the car all on its own.
No image of the hill is shown.

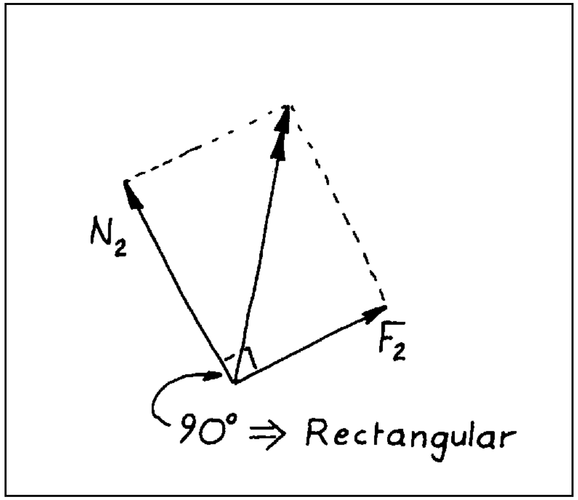


Where once were contacts with the ground
Force arrows in their place are found.

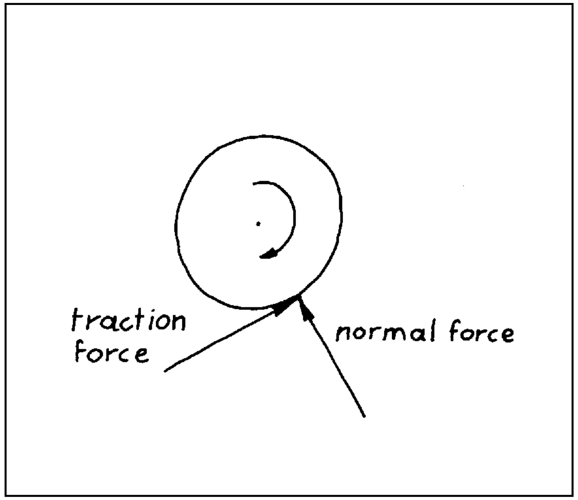


The weight acts at the 'c of g'
("Assume you'll know where this will be").

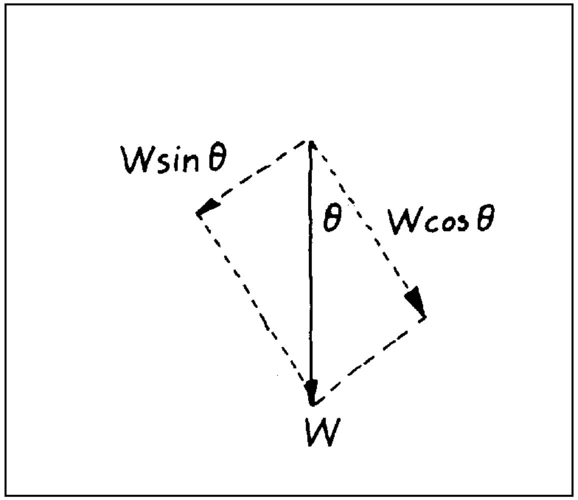
RECTANGULAR COMPONENTS PROVE VERY USEFUL



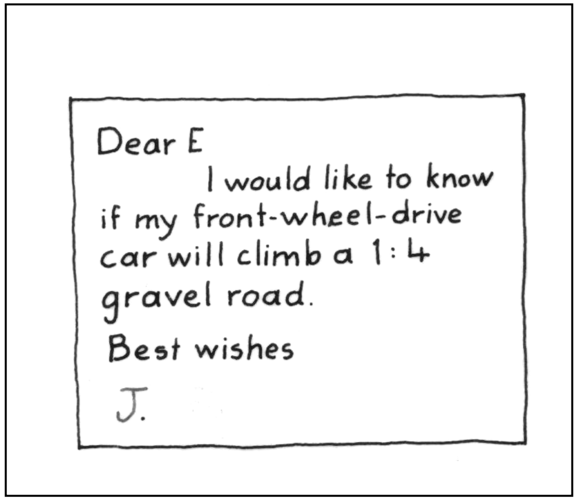
The forces at the front wheels are
'components', called 'rectangular'.



F_2 is traction; it's what will
Pull Jamie's car along the hill.



Components of the weight they'll use.
Along the hill is what they choose.

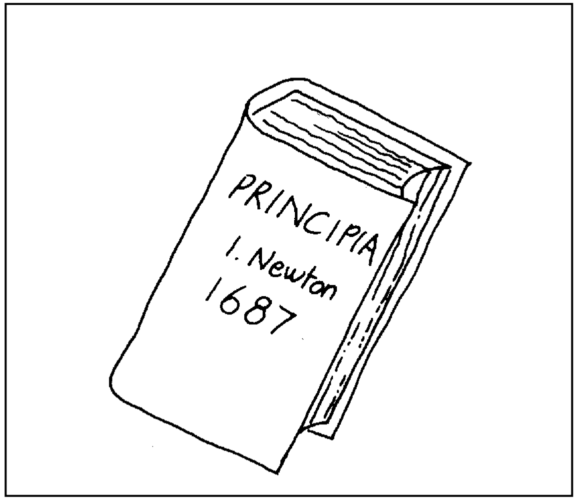


"Let's find how big F_2 must be
To pull the car up steadily."

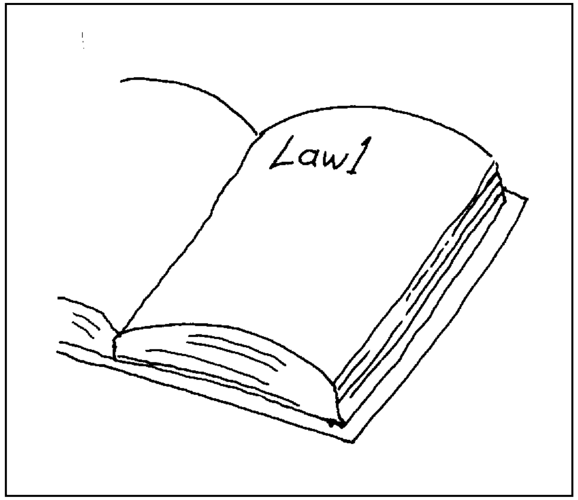
Lee checks with Jamie, "Does the FBD make sense now?" "More or less," Jamie answers, "but exactly how do you know what force arrows to use. I mean there are two on the front wheel and only one at the back." Kim buts in, "It's a mixture of intuition and convention. You'll find conventions listed in books. There's only one arrow at the back because we're assuming free rolling there. At the front there are two arrows because we're using components to show the traction force that pulls the car up the hill, and the normal force at right angles to it." "Are you ok on components of a vector?" asks Lee. "Yes," says Jamie, confidently, "It's when you split up a vector using a parallelogram." "Right," says Lee, "Only we'll use a rectangle instead – 'rectangular components'." Kim adds, "We'll split up the weight too, into one component along the hill and one at right angles to it." Jamie asks, "So, next we'll find out how big the traction force must be to pull my car up the hill?" A mobile phone rings and the twins have to leave. "We'll give you a book to read, Jamie. Check out equilibrium and we'll get back to you."

SIR ISAAC EXPLAINS EQUILIBRIUM

Back at home Jamie looks at the section of the book on equilibrium. Sir Isaac Newton features a lot. His first law makes immediate sense: 'Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it.' The book explains that to use Newton's concept of equilibrium you add force components in any direction you choose and put the result equal to zero. In Jamie's case you can do this for two different directions and get two equations of equilibrium, so you can solve for two unknowns. When doing all this, the book is as keen on using Free Body Diagrams as the engineering twins are, and it has all sorts of rules for them. Jamie thinks for a moment about trying to work out the traction force unaided, but decides not to bother, "Might as well wait for the engineering twins to turn up again. They won't be long. I'll go for a drive in my new car instead." It had just started to rain after a long dry spell, and pulling away from rest on a slippery hill Jamie notices wheelspin. "It must be that weight transfer," thinks Jamie.

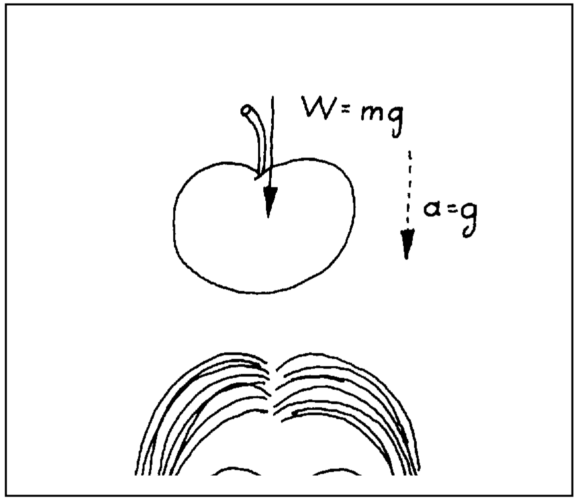


"Sir Isaac Newton, come, please, come!
Teach Jamie equilibrium."



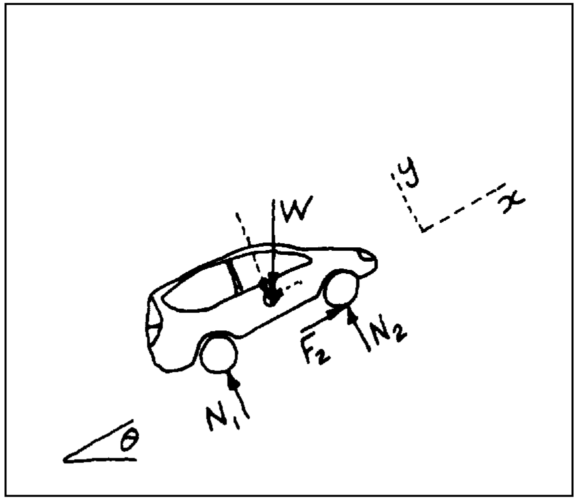
"In equilibrium, I say,
The forces balance, every way."

"Things keep at rest, I then define,
Or steady speed in one straight line."

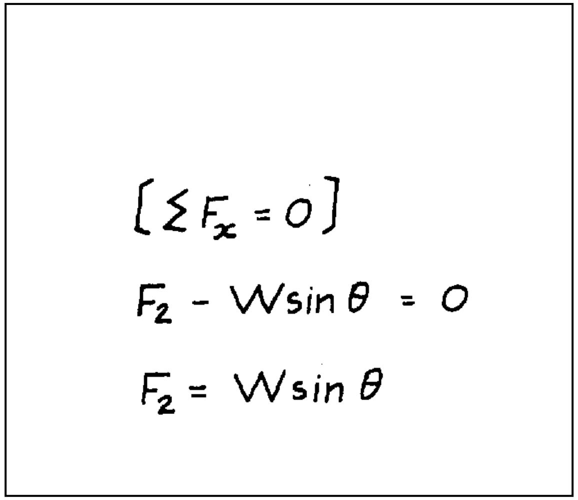


"You'll solve your problem, you will find,
With this great truth from my great mind."

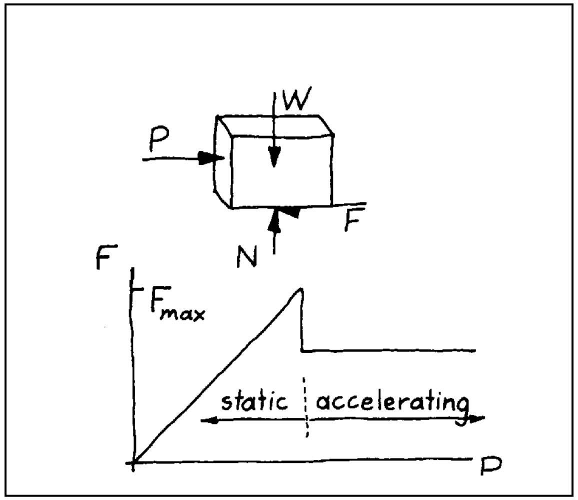
THE TWINS USE EQUILIBRIUM AND JAMIE DISCOVERS A NEW QUESTION



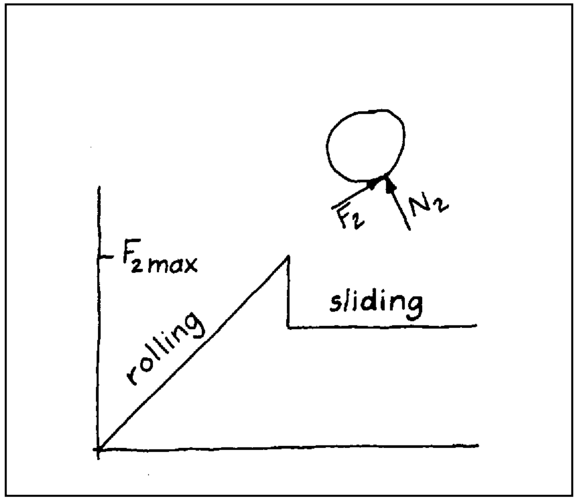
"Find 'sum of forces up the hill'.
For equilibrium it's nil."



A little maths gives them F_2,
But there is still more work to do.



F_2 describes a friction force.
There is a maximum of course.

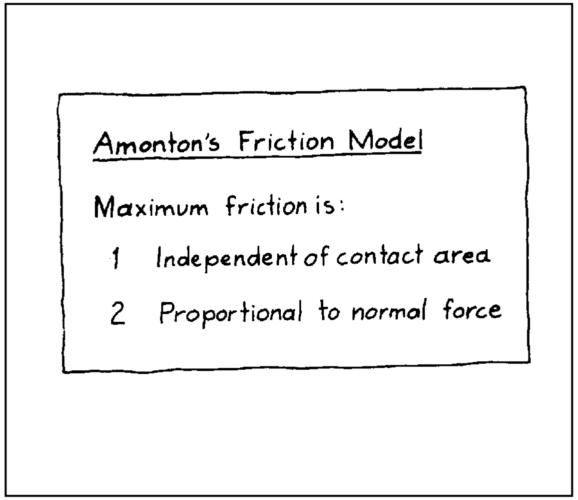


They need to know if what they've found
Fits nicely in the upper bound.

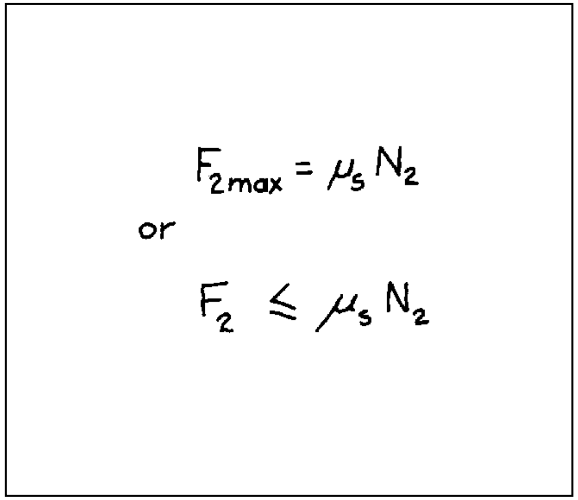
"You could have worked out the traction force for yourself, Jamie," says Lee. "It's easy. Couldn't you find the component of the weight down the hill?" "I had better things to do," says Jamie. Lee, back on task, starts writing out the equations and slowly says, as her pencil writes '$\Sigma F_x = 0$' on the sketch-pad , "Sum of forces in the *x* direction equals zero." Kim points out that the *x* direction is given by the axes they had previously added to the FBD. Underneath that line Lee writes the two force components that are to be summed, with zero on the other side, then the final line gives the answer. "That's neat," says Jamie, "but don't we need to find the *maximum* friction force we can get." "Right," says Kim. "That's next." He puts a box on the table and pushes it sideways until it slides. "When I do this I have to push hard to get it moving, then it's easier once it's sliding." Lee adds "When you brake your car it stops more quickly at the end, because as the wheel stops rotating the friction goes from sliding to static." "But we need numbers, not just ideas," says Jamie. "Amontons and Coulomb can help with that," says Kim.

TWO NEW PALS PROVIDE A KEY TO FRICTION

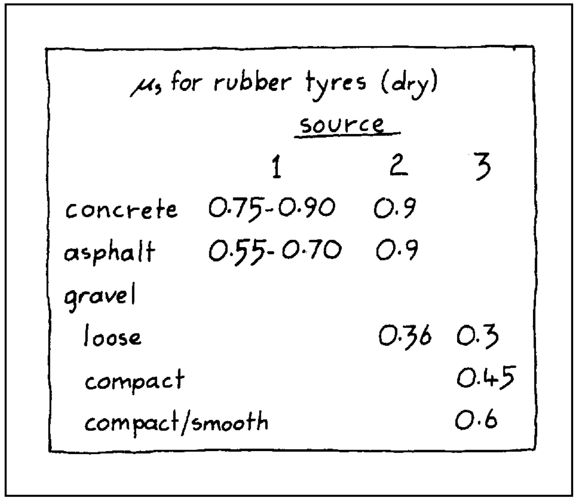
"Who are they?" asks Jamie. "We've got to go" say the twins. "But use our computer if you want to find out." From a quick Google, Jamie finds that they were two Frenchmen. Amontons did tests in the late 1600s and proposed two laws of friction: maximum friction is (i) proportional to normal force ($F_2 \leq \mu_s N_2$), and (ii) independent of the size of the contact. Both laws are approximations. Coulomb did more tests and worked out a third law: kinetic (sliding) friction tends to be less than static friction. Coefficients of friction, μ, vary a lot, but if you can't measure them you can get estimates from books or the web. "If maximum friction is proportional to normal force we'll need N_2," thinks Jamie, and starts using equilibrium again. $\Sigma F_y = 0$ is no good (two unknowns); horizontal force equilibrium still gives too many unknowns. "I know," thinks Jamie, "I'll combine the two and add in the equation for traction force." The working had just lead to 0 = 0 when the engineering twins return. "I can see your problem," says Lee, "You've got two new unknowns to find and just one valid extra equation. You need another one."



"Dear Amontons and Coulomb, how
Can we work out that limit now?"

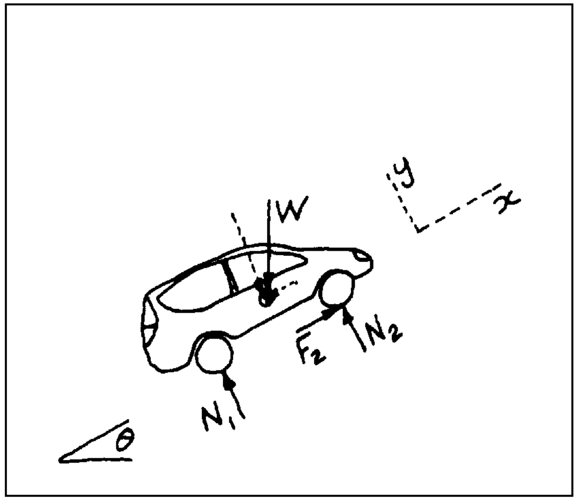


"The most the friction force can be
Is N_2 times μ_s you see."



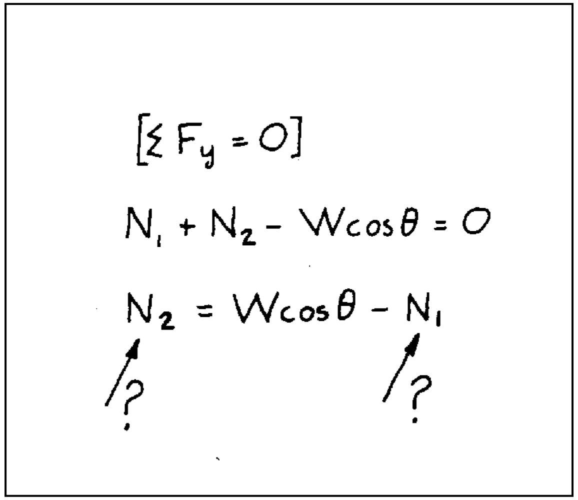
	source 1	2	3
concrete	0.75-0.90	0.9	
asphalt	0.55-0.70	0.9	
gravel			
loose		0.36	0.3
compact			0.45
compact/smooth			0.6

μ_s you measure, or you get
It roughly from the internet.

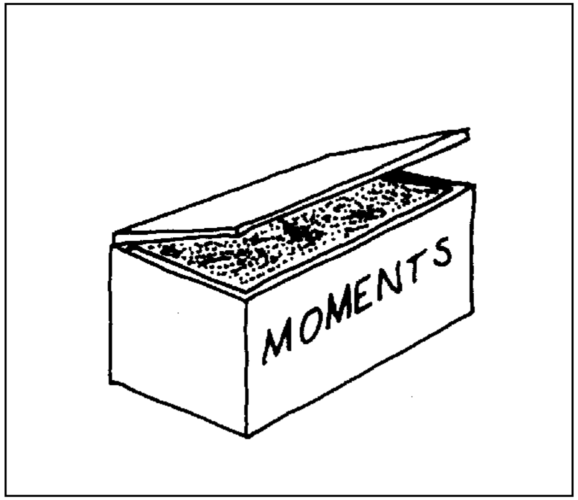


So force N_2 they need to find
But at first glance they're in a bind.

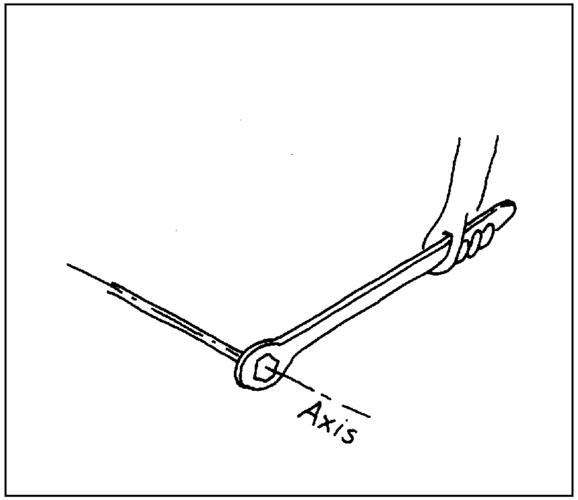
JAMIE FINDS OUT ABOUT MOMENTS



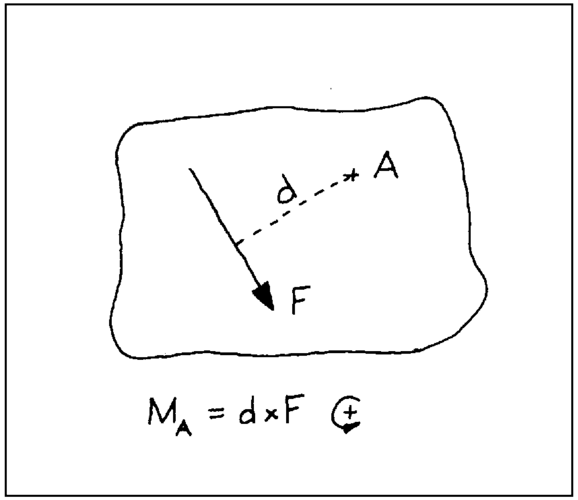
More summing forces will not do.
Equations: one, but unknowns: two.



"You need to know," the twins then say,
"That taking 'moments' is the way."



"A moment tends to make things turn
About an axis, you must learn."

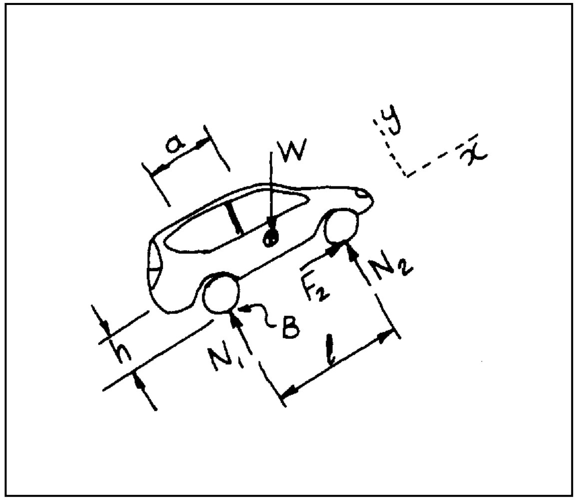


And, force times moment arm supplies
A measure of a moment's size.

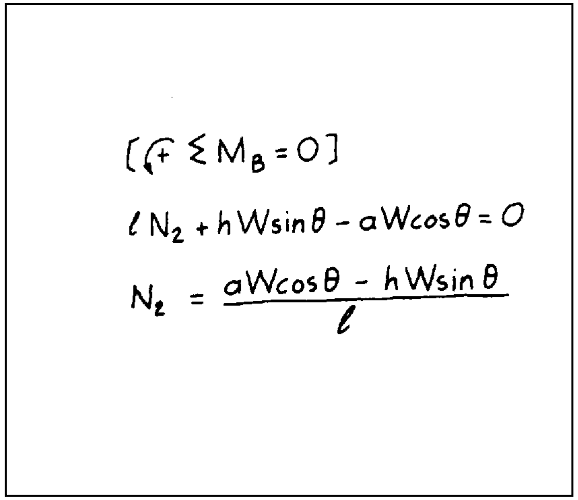
Lee continues, "We've got a two-dimensional force system, so force equilibrium gives only two equations. Try to get more, and you just find things that aren't new." "Like 0 = 0," jokes Jamie, and continues, "But Kim, where do we get the extra equation?" Kim explains, "We need to make sure the car doesn't tend to rotate, so we need to balance out twisting effects. The twisting effect of a force is called a moment – don't know why." "How do you measure it?" asks Jamie. "I'll need the sketch pad for this," says Kim, and draws a strange shape with a force acting on it. "See this point *A*," he says, "it's the end view of an axis, and the twisting effect of force *F* about the axis is *F* times this distance *d* – the moment arm." "I see," says Jamie, "it's the perpendicular distance from *A* to the line of action of the force. Seems reasonable: twice the force – twice the moment; twice the distance – twice the moment." Lee adds, "Of course, if a force acts through the point it doesn't generate any moment at all." "No moment arm," says Jamie. "Right," says Lee, "and we'll use that when we choose an axis for taking moments for equilibrium."

JAMIE GETS AN ANSWER

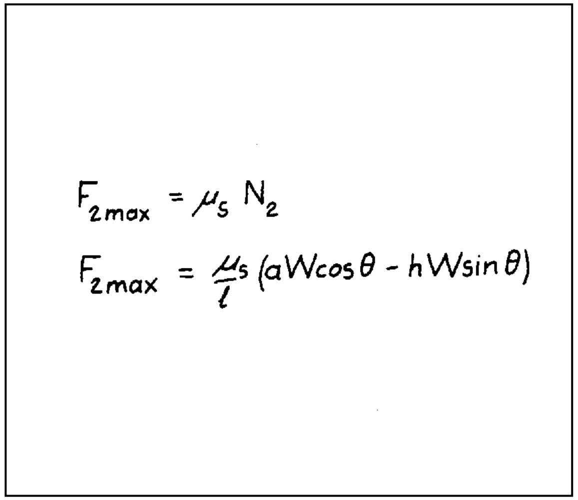
Lee explains, “For our final equilibrium equation we add up all the moments about any axis we choose and put the result equal to zero. It’s good to show the dimensions you use on the FBD as a record.” “So we’ll get three equations: two for forces; one for moments,” says Jamie. “That’s true,” says Kim, “although you can have one for forces and two for moments, or even three for moments if you like – but you only get three altogether.” “Anyway,” says Lee, “Back to our FBD. We’ll take moments about point B – it knocks out N_2 and F_1, see.” Lee’s pencil moves across the page. “Sum of moments about point B equals zero, positive anticlockwise,” says Lee as she writes the first statement. “Does it have to be anticlockwise?” asks Jamie. “No,” says Kim. “You’ve just got to stick with whatever you decide.” A few lines later there is an expression for N_2. “Great!” says Jamie. “Now $\mu_s N_2$ will give us the maximum traction force.” “Here’s the pencil, Jamie,” says Lee. “You do it.” They get an expression for F_{2max}, ready to compare it with the traction force, F_2, they had found the expression for earlier. If F_2 is less than F_{2max} the car can climb the hill.



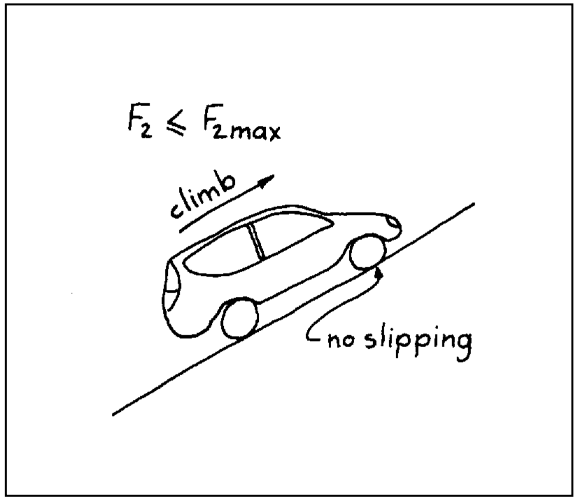
For equilibrium, they will
Use ‘sum of moments’ equals nil.



By summing moments at point B
They get N_2 quite easily.

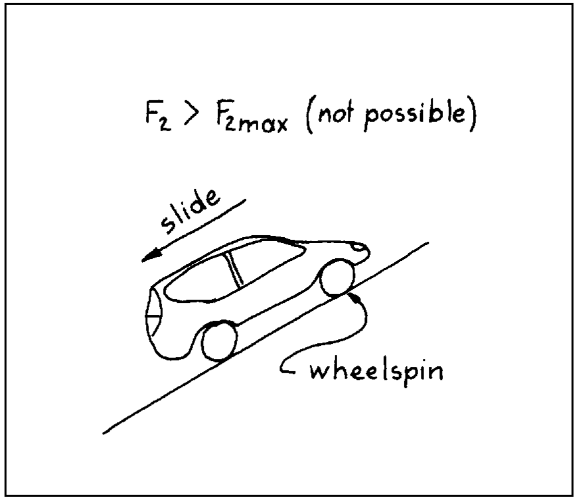


The twins explain “$\mu_s N_2$
Gives maximum F_2 to you.”

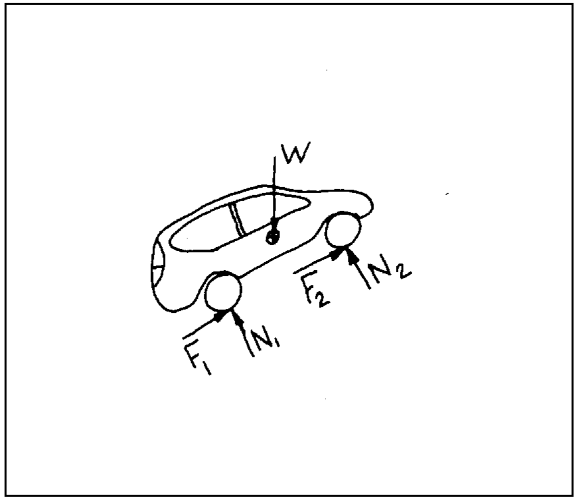


“If max is more than what you need
Your car can climb at steady speed.”

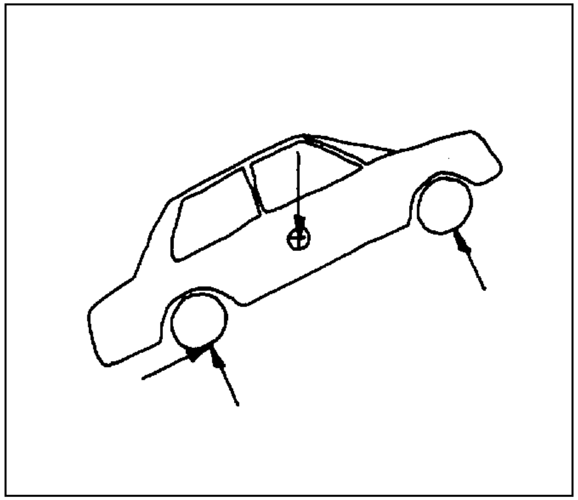
JAMIE ARRIVES AT PRACTICAL INSIGHTS



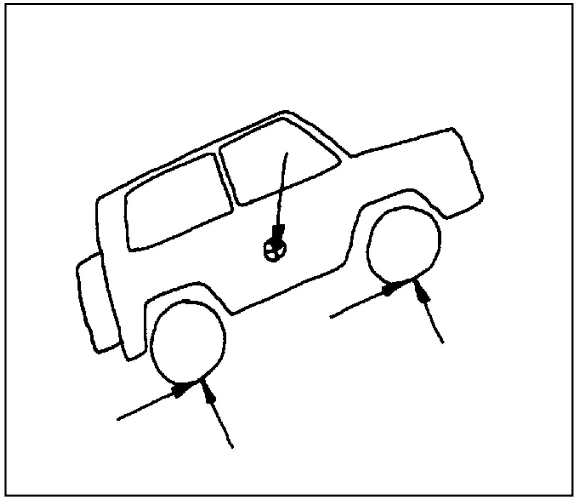
If max is less, the car slides down
And Jamie's face will wear a frown.



With luck they'll find the four-wheel brake
Can stop the slide, for safety's sake.



Some rear wheel drives climb more no doubt.
Weight transfer to the rear helps out.

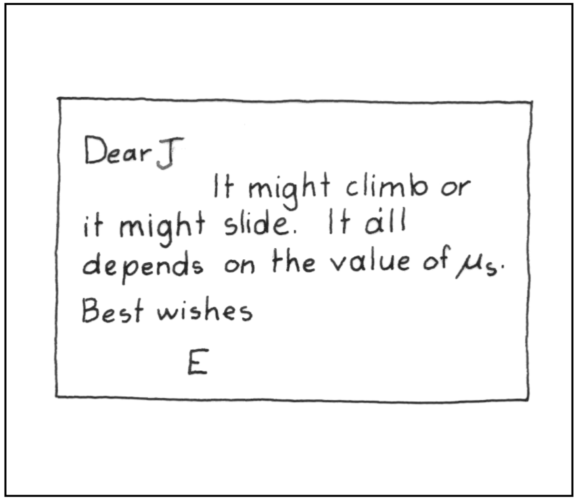


A four wheel drive is better still,
For it can climb a steeper hill.

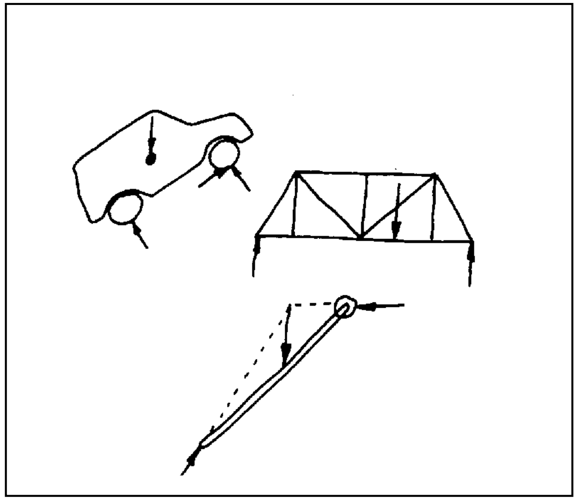
"Let's try numbers now," suggests Jamie. "We can measure the wheelbase and guess the rest." "Where do you think the centre of gravity is Lee?" asks Kim. Lee replies, "With just the driver in the car I'd say there'd be 55% of the weight on the front wheels when it's on the level, so the cg would be 55% of the track in front of the rear wheels." "How did you work that out?" asks Jamie, impressed. Kim replies, "It's a trade secret – more Statics." "How about the height of the cg?" asks Jamie. "We'll eyeball that one," says Kim. "How does 550mm seem to you?" They measure the wheelbase at 2400mm, put the rest of the data into the equations and find that the coefficient of friction needed for Jamie's car to climb a one-in-four hill is 0.56. "Would gravel roads give that?" asks Jamie. "I doubt it," says Kim, "but if you started sliding backwards you could use the footbrake and get resistance at the back wheels too." "Would a rear wheel drive car be better?" asks Jamie, "because of the weight transfer to the back?" 'Possibly," says Lee, "It depends on weight distribution on the level." "Best of all is a four wheel drive," says Kim.

JAMIE BEGINS TO GET ENGINEERING EYES

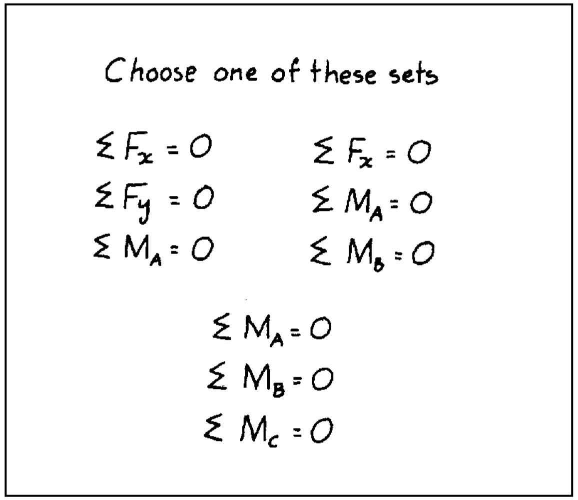
Next day, back at home, Jamie thinks over what the engineering twins had demonstrated. It was all very clever, but had it really answered the question? With their method you can analyse the situation, but to get answers you need to know where your car's centre of gravity is and the coefficient of friction between the tyres and the road, and you can't easily get either of them. But still, the method does show you what things are important and what you can do if you get into trouble, like sliding backwards. So perhaps Jamie's questions were answered, in a way. Then there were Kim's words as Jamie drove off: "Hey, Jamie, you'll be seeing the world through engineering eyes next." "That was a strange idea," thinks Jamie, then realises that the world has indeed been looking different. Free Body Diagrams keep coming to mind, uninvited, and Newton's first law keeps raising its head. Then there are the insights into friction from the two Frenchmen. "Soon I'll end up thinking like the twins," muses Jamie, "Scary. And now I've changed my thinking I can't go back. The world will look different forever."



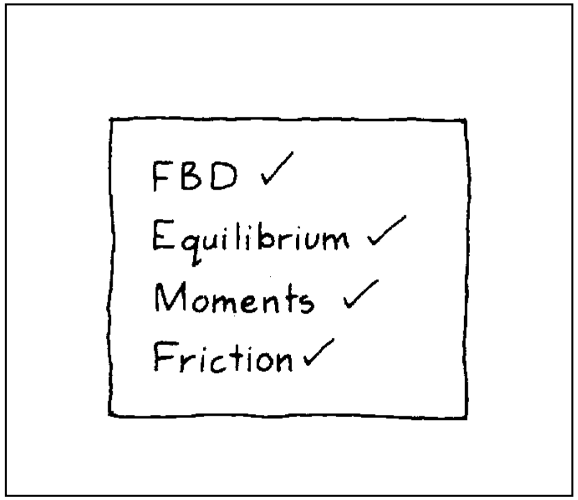
So Newton, Amontons et al
Help solve a problem for our pal.



But Jamie finds an FBD
Will pop up unexpectedly.



And equilibrium, somehow,
Is fixed in Jamie's mind by now.



Then Jamie says, with some surprise,
"I'm getting engineering eyes!"

The End